

Chess endgames: data and strategy

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CHESS ENDGAMES: DATA AND STRATEGY

J.A. Tamplin, G.M^cC. Haworth

jat@jaet.org; guy_haworth@hotmail.com, <http://www.jaet.org/jat/>

Abstract While Nalimov's endgame tables for Western Chess are the most used today, their Depth-to-Mate metric is not the only one and not the most effective in use. The authors have developed and used new programs to create tables to alternative metrics and recommend better strategies for endgame play.

Keywords: chess, conversion, data, depth, endgame, goal, move count, statistics, strategy

1. Introduction

Chess endgames tables (EGTs) to the 'DTM' Depth to Mate metric are the most commonly used, thanks to codes and production work by Nalimov (Nalimov, Haworth, and Heinz, 2000a,b; Hyatt, 2000). DTM data is of interest in itself, even if *conversion*, i.e., change of force, is usually adopted as an interim objective in human play. However, more effective endgame strategies using different metrics can be adopted, particularly by computers (Haworth, 2000, 2001). A further practical disadvantage of the DTM EGTs is that, with more men, DTM increases and file-compression becomes less effective.

Here, we focus on metrics DTC, DTZ¹ and DTZ₅₀²; the first two were previously used by Thompson (1986, 2000) and Wirth (1999). New programs by Tamplin (2001) and Bourzutschky (2003) have enabled a complete suite of 3-to-5-man DTC/Z/Z₅₀ EGTs to be produced.

Section 2 outlines these two new algorithms. Sections 3 to 5 review the new DTC, DTZ and DTZ₅₀ data tabled in the Appendix. Finally, improved endgame strategies are recommended for the 50-move context.

¹ DTC \equiv *Depth to Conversion*, i.e., to force change and/or mate.

DTZ \equiv *Depth to (Move-Count) Zeroing (Move)*, i.e., to P-push, force change and/or mate.

² DTZ_k \equiv DTZ, but *draw* if the 'win' can be pre-empted by a *k*-move draw claim.

2. New Approaches to EGT Generation

Below we briefly describe two new approaches to EGT generation. The first one is described adequately in the literature; the second so far not.

2.1 Tamplin's Wu-Beal Code

Tamplin (2001) combined the Wu-Beal (2001a,b) algorithm with Nalimov indexing in a new code whose objectives were primarily Nalimov-compatibility, simplicity, maintainability and portability. Most pawnless 3-to-5-man DTC EGTs were generated, the new code including an inverse-index function mirroring Nalimov's index function.

2.2 Bourzutschky's Modified-Nalimov Code

Bourzutschky (2003) modified Nalimov's DTM-code to enable it also to generate EGTs to metrics DTC_k and DTZ_k . This involved generalising some DTM-specific aspects of the algorithm, as well as the obvious changes to the iterative formula for deriving depth. For DTC, the code retains the efficiencies of the DTM-code while requiring maxDTC rather than maxDTM cycles. Because EGT generation to the DTZ metric has not yet been implemented generically as a sequence of sub-EGT generations, each based on a fixed pawn structure, this is not the case for DTZ_k computations. These can also require somewhat more than DTC cycles but the difference is insignificant.

3. The DTC Data

DTC EGTs are interesting, not only for completeness, but because *conversion* is an intuitively obvious objective and the DTC EGTs document precisely the phase of play when the material nominated is on the board.

The remaining 3-to-5-man DTC EGTs were generated. Table 1 in the Appendix lists for each endgame the number of positions of maxDTC, wtm/btm and 1-0/0-1. The ICGA (2003) website provides further data, including %-wins, illustrative maxDTC positions and DTC-minimaxing lines. Because there are many wins in 1, the *% of positions won* does not characterise well the presence of wins in an endgame. Similarly, maxDTC is not a good indicator. We therefore suggest a new characteristic,

$$\text{Win-Presence} \equiv \%_of_positions_won \times (\text{Average DTC of Win})$$

This is not unduly affected by the usual peak of wins in 1 or by the long tail of deep wins, and is in fact related to the number of moves for which a win is present on the board.

3.1 A Review of the DTC Data

A first housekeeping point to be made is that this data often differs from Wirth's data (Wirth and Nievergelt, 1999; Tamplin, 2003). The explanation is simple. First, Wirth has exactly one representative of each equivalence class of positions, including the harder case of both Kings being on a1-h8. Nalimov would count $\{\text{wKc3Qb3(c2)/bKa1}\}$ as two positions rather than one.

Second, Wirth's code, based on the inherited RETROENGINE, assumes that all conversions are effected by the winner. This is not so: the loser is sometimes forced to convert to loss, e.g., $\{\text{wKe1Qb1Rf1/bKa1}\}$, in which case Wirth's depth is too great by one.

Tamplin's (2003) and Bourzutschky's (2003) codes both measure depth consistently in *winner's moves*. Also, they do not allow 'realistic' but voluntary conversions, e.g., $\{\text{wKe1Qf1Rb1/bKa1}\}$, by the loser, a feature of Thompson's original DTC EGT code (Thompson, 1986) which chose to move to the position with greatest DTC even if a capture was involved.

The sub-6-man compressed DTC EGTs are 62.1% the size of the DTM EGTs, usefully saving 2.8GB disc space.

The $\text{maxDTC}=114$ wins in KNNKP and KQPKQ are already known. KBNK wtm scores the highest in *Win-Presence* terms: $\text{maxDTC} = 33$, average DTC = 24.68 and 99.51% of positions are 1-0 wins.

4. The DTZ Data

The DTZ metric is necessary if the length of the current phase of play is to be *guarded* in the context of chess' k -move rule, k currently being 50. It was used pragmatically by Thompson (1986) to compute the KQPKQ and KRPKR EGTs when RAM was relatively scarce.

Bourzutschky (2003) generated some DTZ EGTs where $\text{maxDTZ} > 50$ and Tamplin (2003) completed the sub-6-man DTZ EGT suite. The computation continues to be a major feat as it cannot currently use Nalimov's bitvector-based algorithm which reduces RAM requirements by a factor of 4 to 16.

Table 2 in the Appendix lists the results which differ from the DTC data. KNNKP with $\text{maxDTZ} = 82$ features the deepest endings. DTZ EGTs are commendably compact relative to DTM and DTC EGTs. The KPPPK wtm DTZ EGT is an extreme example, being only 2% the size of the DTM EGT. In total, the sub-6-man compressed DTZ EGTs are 52.9% the size of the DTM EGTs, usefully saving some 3.5GB of disc space.

5. The DTZ₅₀ Data

Bourzutschky (2003) and Tamplin (2003) also generated DTZ₅₀ EGTs, not only for those cases where maxDTZ > 50, but for endgames directly or indirectly dependent on these as illustrated in Figure 1. The DTZ₅₀ metric rates as wins only those positions winnable against best play given the 50-move rule. In Figure 1, endgames for which EZ and EZ₅₀ are potentially but not actually different are in brackets, and dotted lines indicate that no 50-move impact emanates from or feeds back to them.

The sub-6-man compressed DTZ₅₀ EGTs are 49.8% the size of the DTM EGTs. Table 3 in the Appendix lists 3-to-5-man DTZ₅₀ EGT data for endgames where DTZ₅₀ ≠ DTZ and Table 7 gives examples of positions affected. Table 6 summarises 50-move impact, minimal for KNPQ, considerable for KBBKN and KNNKP.

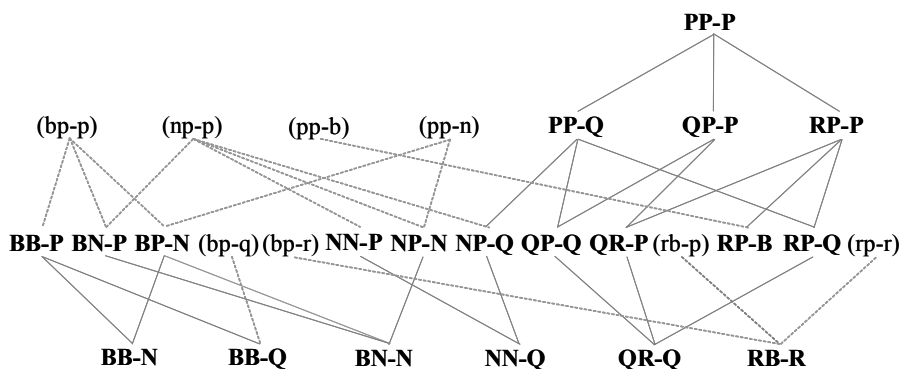


Figure 1. Endgames with EZ₅₀ ≠ EZ.

If $KwKb$ is an endgame with wtm and btm 1-0 wins impacted by the 50-move rule, $KwxKb$ and $KwKby$ are also impacted by the rule. This observation, coupled with Thompson's DTC results (Tamplin and Haworth, 2001) and the DTM results of Nalimov (Hyatt, 2000) and Bourzutschky (2003) indicate that many 6-man endgames are affected. Tamplin (2003) has computed some of these 6-man endgames' EGTs to the DTZ and DTZ₅₀ metrics.

In contrast with KNNKP, KBBKNN has the majority of its wins frustrated, and few wins can be retained by deeper strategy in the current phase. There are significant percentages of frustrated 0-1 wins in KBBBKQ, and of delayed 1-0 wins in KBBBKN and KBBNKN.

Elsewhere, there is only the merest hint of the 50-move impact that might follow and we would expect that hint to become fainter as the number of men increases.

6. Endgame Strategies

Let dtx be the depth by, and Ex an EGT to, the metric DTx . Let Sx^- be an endgame strategy minimising dtx , e.g., SZ^- , or SZ_{50}^- , and let Sx^+ be a strategy maximising dtx . Further, let SZ° be an endgame strategy *guarding* the length of the current phase in the context of a k -move rule and a remaining *mleft* moves before a possible draw claim. By definition, if $dtx > mleft$, $Sx^\circ \equiv Sx^-$.

Let $Ss_1s_2s_3$ be an endgame strategy using strategies Ss_1 , Ss_2 , and Ss_3 in turn to subset the choice of moves, e.g., $SZ^\circ Z_{50}^- M^- Z^-$ which safeguards current phase length and 50-move wins, and then minimises dtm and dtz in turn.

As conjectured by Haworth (2000), KQPKQ and KBBKNN provide positions where all combinations of SC^- , SM^- and SZ^- *fail* to safeguard a win available under the 50-move rule: the examples here were found by Bourzutschky (2003). Similar positions for other endgames were found by Tamplin (2003). Some strategy-driven lines are listed in Appendix 1 after Table 5.

6.1 New Endgame Strategies

SZ_{50}^- wins any game winnable against best play under the 50-move drawing rule. Here, we suggest ways to finesse wins against fallible opposition. If the current phase of play is not unavoidably overlong, strategy $SZ^\circ Z_{50}^- Z^-$, effectively $SZ^\circ Z^- \equiv SZ^-$, completes it without a draw claim.

For positions where DTZ_{50} indicates *draw*, the table EZ_{50} can be supplemented by the position's DTR^3 value. Let this hybrid table be EH_{50} , implicitly defining metric DTH_{50} . Note that EZ_{50} is visible within EH_{50} . Since the intention is to use EH_{50} only in conjunction with EZ , let the table $E\delta(H/Z)_{50}Z \equiv \{\delta(DT(H/Z)_{50}, DTZ)\}$, giving a compact encoding⁴ of $E(H/Z)_{50}$ decodable with the use of EZ . With $E\delta Z_{50}Z = \Phi$ if $EZ_{50} \equiv EZ$, sub-6-man compressed $E\delta Z_{50}Z$ EGTs are only 0.7% the size of the DTM EGTs.

The strategy $SZ^\circ H_{50}^-$ guards the length of the current phase, wins all games which are wins under the 50-move rule, and minimaxes DTR , but only tactically, when the 50-move rule intervenes.

In position NN-P3, $SZ^\circ H_{50}^-$ makes the optimal move-choice⁵. In contrast, $SZ^\circ Z_{50}^-$ can, and $S\sigma$ ($\sigma \equiv C^-, M^-, Z^-, Z^\circ Z_{50}^- Z^-$) does, concede DTR depth. However, $SZ^\circ H_{50}^-$ has two flaws, the first being a major one. It can draw by repeating positions, e.g., position NN-P4⁶. $SZ^\circ H_{50}^-$ should therefore be augmented by as deep and perceptive a forward search as possible, denoted here by ** as in $SZ^\circ H_{50}^{-**}$.

³ $DTR \equiv \text{Depth by The Rule}$ (Haworth, 2000, 2001), i.e. the minimum k s.t. DTZ_k is a win.

⁴ We chose $0 \equiv \text{"EZ code = Ex}_k \text{ code"}$, $1 \equiv \text{"new EZ}_{50} \text{ draw"}$, $\delta+1 \equiv "0 < DTx - DTZ = \delta"$.

⁵ $SZ^\circ H_{50}^- - SH_{50}^+$: 1. Nb1+! Ka4! White retains $DTR=51$ and converts in 31 moves.

⁶ NN-P4, $SZ^\circ H_{50}^- - SH_{50}^+$: 1. Nd5+? Kc4! 2. Ndc3 Kb4! {NN-P4 repeated}.

If position NN-P4, with $dtz_{51} = 25$, has just 25 moves left in the phase, it also shows $SZ^{\circ}H_{50}^-$ failing to achieve minimal DTR. The move Nd5+ is optimal for SH_{50}^- but DTZ₅₁-suboptimal, a fact not visible in the EGT EH₅₀. After Nd5+, SZ^o limits the move choice and puts a DTR of 51 out of reach. Again, forward search helps, this time aiming to control DTR.

Any strategy can be sharpened by the opponent sensitivity of an adaptive, opponent model (Haworth, 2003; Haworth and Andrist, 2003).

7. EGT Integrity

All EGT files were given md5sum signatures to guard against subsequent corruption. The EGTs were checked for errors in various ways.

- DTx EGTs {Ex}, $x = C, Z$ and Z_{50} , verified by Nalimov's standard test.
- consistency of the {E(C/M/Z)} EGTs confirmed
theoretical values found identical with $dtm \geq dtc \geq dtz$.
- DTC EGT statistics were also found compatible with those of Wirth.
- consistency of the {EZ₅₀} and {EZ} EGTs confirmed
linear checks confirm $EZ_{50} \equiv EZ$ except for known subset,
values identical with $dtz_{50} \geq dtz$, or 'EZ' win/loss an 'EZ₅₀' draw.

8. Summary

This paper records the separate initiatives of Tamplin (2003) and Bourzutschky (2003) in creating new codes capable of generating non-DTM EGTs. It also reviews the new DTC/Z/Z₅₀ data produced by the combination of these codes. The DTC, DTZ and DTZ₅₀ EGTs (EC, EZ and EZ₅₀) are increasingly compact compared to the DTM EGTs, an incidental but practical benefit with 3-to-6-man DTM EGTs estimated to be 1 to 2 TB in size.

Together, the sub-6-man compressed EZ and E δ Z₅₀Z EGTs are 53.6% the size of the EM EGTs. To date, the equivalent 6-man EGTs are 63.8% the size of their EM EGT counterparts but these do not yet involve Pawns

Although the computation of DTR data remains a future challenge, table EZ₅₀ may in principle be augmented by DTR values where $dtr > 50$ to give table EH₅₀. This table may be used to minimise dtz_{50} when $dtz_{50} \leq 50$, and to minimax dtr with the assistance of forward-search when $dtr \geq 50$.

Clearly, there are more effective and efficient endgame strategies than the commonly used SM⁻. It is recommended that $SZ^{\circ}M^-$, $SZ^{\circ}Z_{50}^-Z^{(*)}$, $SZ^{\circ}Z_{50}^-ZH_{50}^{(*)}$, $SZ^{\circ}H_{50}^-*$ and perhaps other strategies are considered, and that the EZ, E δ Z₅₀Z and E δ H₅₀Z EGTs are made available to enable their use.

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Appendix: Chess Endgame Data and Examples

Endgame			DTC Metric							
			# of maximal positions				max depths, moves			
			1-0		0-1		1-0		0-1	
Name	GBR	# w-b	wtm	btm	wtm	btm	wtm	btm	wtm	btm
KBK	0010.00	3 2-1	0	0	0	0	—	—	—	—
KNK	0001.00	3 2-1	0	0	0	0	—	—	—	—
KPK	0000.10	3 2-1	3	2	0	0	19	19	—	—
KQK	1000.00	3 2-1	1	8	0	0	10	10	—	—
KRK	0100.00	3 2-1	139	433	0	0	16	16	—	—
KBKB	0040.00	4 2-2	52	14	14	52	1	0	0	1
KBKN	0013.00	4 2-2	2	1	1	5	1	0	0	1
KBKP	0010.01	4 2-2	104	28	6	14	1	0	5	6
KNKN	0004.00	4 2-2	5	1	1	5	1	0	0	1
KNKP	0001.01	4 2-2	29	7	3	3	7	6	12	13
KPKP	0000.11	4 2-2	1	1	1	1	14	14	14	14
KQKB	1030.00	4 2-2	980	4,837	0	0	12	12	—	—
KQKN	1003.00	4 2-2	5	19	0	0	19	19	—	—
KQKP	1000.01	4 2-2	1	1	20	20	26	26	1	2
KQKQ	4000.00	4 2-2	5	3	3	5	10	9	9	10
KQKR	1300.00	4 2-2	2	11	55	291	31	31	2	3
KRKB	0130.00	4 2-2	29	1	0	0	18	18	—	—
KRKN	0103.00	4 2-2	2	2	1	4	27	27	0	1
KRKP	0100.01	4 2-2	28	42	3	3	16	16	10	11
KRRR	0400.00	4 2-2	59	111	111	59	4	3	3	4
KBBK	0020.00	4 3-1	16	59	0	0	19	19	—	—
KBNK	0011.00	4 3-1	144	436	0	0	33	33	—	—
KBPB	0010.10	4 3-1	2	8	0	0	21	21	—	—
KNNK	0002.00	4 3-1	77	15	0	0	1	0	—	—
KNPK	0001.10	4 3-1	24	32	0	0	22	22	—	—
KPPK	0000.20	4 3-1	62	21	0	0	16	16	—	—
KQBK	1010.00	4 3-1	2,411	14,012	0	0	6	6	—	—
KQNK	1001.00	4 3-1	4,932	23,203	0	0	7	7	—	—
KQPK	1000.10	4 3-1	75	175	0	0	7	7	—	—
KQQK	2000.00	4 3-1	3,280	13,005	0	0	3	3	—	—
KQRK	1100.00	4 3-1	44	158	0	0	4	4	—	—
KRBK	0110.00	4 3-1	1	6	0	0	12	12	—	—
KRNK	0101.00	4 3-1	324	1,017	0	0	12	12	—	—
KRPK	0100.10	4 3-1	376	1,885	0	0	8	8	—	—
KRRK	0200.00	4 3-1	68	287	0	0	5	5	—	—
KBBKB	0050.00	5 3-2	503	6	141	546	6	6	1	2
KBBKN	0023.00	5 3-2	34	53	44	222	66	66	0	1
KBBKP	0020.01	5 3-2	34	69	5	11	21	21	8	9
KBBKQ	3020.00	5 3-2	248	58	74	15	4	3	71	71
KBBKR	0320.00	5 3-2	26	7	2	6	7	6	8	9
KBNKB	0041.00	5 3-2	28	19	133	514	13	12	1	2
KBNKN	0014.00	5 3-2	2	1	104	533	77	76	0	1
KBNKP	0011.01	5 3-2	1	2	523	535	26	26	8	9
KBNKQ	3011.00	5 3-2	79	1	22	4	5	5	42	42
KBNKR	0311.00	5 3-2	127	23	4	2	6	5	12	13
KBPKB	0040.10	5 3-2	14	14	508	1,524	40	39	2	3
KBPKN	0013.10	5 3-2	16	6	23	86	42	42	3	4
KBPKP	0010.11	5 3-2	92	52	27	23	53	53	6	7
KBPQK	3010.10	5 3-2	30	30	3	2	4	3	42	42
KBPKR	0310.10	5 3-2	76	53	5	6	13	12	20	21

Table 1a. Chess Endgames: 3-to-5-man DTC data.

Endgame			DTC Metric							
			# of maximal positions				max depths, moves			
			1-0		0-1		1-0		0-1	
Name	GBR	# w-b	wtm	btm	wtm	btm	wtm	btm	wtm	btm
KNNKB	0032.00	5 3-2	251	82	51	109	4	3	0	1
KNNKN	0005.00	5 3-2	38	18	56	293	7	6	0	1
KNNKP	0002.01	5 3-2	2	4	1	1	114	113	12	13
KNNKQ	3002.00	5 3-2	2,387	465	10	2	1	0	63	63
KNNKR	0302.00	5 3-2	2	1	6	11	3	2	10	11
KNPKB	0031.10	5 3-2	11	3	5	18	31	30	8	9
KNPKN	0004.10	5 3-2	9	2	27	132	48	48	3	4
KNPKP	0001.11	5 3-2	1	6	6	9	33	33	13	14
KNPKQ	3001.10	5 3-2	2	2	1	1	5	4	43	43
KNPKR	0301.10	5 3-2	8	36	7	1	18	18	42	43
KPPKB	0030.20	5 3-2	31	34	14	34	18	17	3	4
KPPKN	0003.20	5 3-2	3	5	21	12	30	29	13	14
KPPKP	0000.21	5 3-2	2	11	66	58	28	28	11	12
KPPKQ	3000.20	5 3-2	14	15	19	8	6	5	30	30
KPPKR	0300.20	5 3-2	1	1	2	3	25	24	25	25
KQBKB	1040.00	5 3-2	220	998	187	645	8	8	1	2
KQBKN	1013.00	5 3-2	74	343	30	153	7	7	0	1
KQBKP	1010.01	5 3-2	5	19	791	789	11	11	1	2
KQBKQ	4010.00	5 3-2	33	1	1	1	30	30	16	17
KQBKR	1310.00	5 3-2	1	6	8,848	52,298	19	19	1	2
KQNBK	1031.00	5 3-2	50	158	28	64	9	9	0	1
KQNBK	1004.00	5 3-2	7	39	31	166	9	9	0	1
KQNBK	1001.01	5 3-2	7	8	928	911	17	17	1	2
KQNBK	4001.00	5 3-2	7	1	1	4	35	35	13	14
KQNBK	1301.00	5 3-2	1	6	15	86	22	22	2	3
KQPKB	1030.10	5 3-2	1,122	4,328	374	1,290	9	9	1	2
KQPKN	1003.10	5 3-2	1	6	3	9	10	10	1	2
KQPKP	1000.11	5 3-2	11,817	39,633	16	16	6	6	2	3
KQPKQ	4000.10	5 3-2	5	13	2	4	114	113	15	16
KQPKR	1300.10	5 3-2	4	20	5,177	26,128	20	20	2	3
KQQKB	2030.00	5 3-2	4	15	0	0	4	4	—	—
KQQKN	2003.00	5 3-2	287	1,411	0	0	4	4	—	—
KQQKP	2000.01	5 3-2	18,995	19,257	140	140	3	3	1	2
KQQKQ	5000.00	5 3-2	2	21	31	152	25	25	6	7
KQQKR	2300.00	5 3-2	2	12	2,383	16,681	14	14	1	2
KQRKB	1130.00	5 3-2	720	2,556	0	0	5	5	—	—
KQRKN	1103.00	5 3-2	234	1,149	36	149	5	5	0	1
KQRKP	1100.01	5 3-2	104,508	131,846	683	683	3	3	1	2
KQRKQ	4100.00	5 3-2	3	31	1	2	60	60	8	9
KQRKR	1400.00	5 3-2	10	54	8,099	56,501	15	15	1	2
KRBKB	0140.00	5 3-2	35	46	251	951	25	25	1	2
KRBKN	0113.00	5 3-2	9	35	106	481	21	21	0	1
KRBKP	0110.01	5 3-2	2	12	4	12	11	11	4	5
KRBKQ	3110.00	5 3-2	1	3	5	4	7	6	41	42
KRBKR	0410.00	5 3-2	28	19	3	14	59	58	3	4
KRNKB	0131.00	5 3-2	3	6	41	89	25	25	0	1
KRNKN	0104.00	5 3-2	5	18	101	468	24	24	0	1
KRNKP	0101.01	5 3-2	65	81	2	2	15	15	10	11
KRNKQ	3101.00	5 3-2	24	5	7	3	9	8	46	46
KRNKR	0401.00	5 3-2	1	1	1	3	33	32	4	5

Table 1b. Chess Endgames: 3-to-5-man DTC data.

DTC Metric											
Endgame				# of maximal positions				max depths, moves			
Name	GBR	#	w-b	1-0		0-1		1-0		0-1	
				wtm	btm	wtm	btm	wtm	btm	wtm	btm
KRPKB	0130.10	5	3-2	11	26	502	1,672	62	62	1	2
KRPKN	0103.10	5	3-2	2	7	4	12	46	46	1	2
KRPKP	0100.11	5	3-2	184	474	17	17	9	9	10	11
KRPKQ	3100.10	5	3-2	5	5	5	1	9	8	78	79
KRPKR	0400.10	5	3-2	33	4	23	80	60	60	6	7
KRRKB	0230.00	5	3-2	1	4	0	0	10	10	—	—
KRRKN	0203.00	5	3-2	215	687	45	184	7	7	0	1
KRRKP	0200.01	5	3-2	16	48	988	988	9	9	1	2
KRRKQ	3200.00	5	3-2	14	4	2	3	15	14	20	20
KRRKR	0500.00	5	3-2	3	15	6,210	43,225	25	25	1	2
KBBBB	0090.00/30	5	4-1	116	345	0	0	10	10	—	—
KBBNK	0021.00	5	4-1	783	2,066	0	0	13	13	—	—
KBBPK	0020.10	5	4-1	3	2	0	0	16	16	—	—
KBNNK	0012.00	5	4-1	22	59	0	0	13	13	—	—
KBNPK	0011.10	5	4-1	9	45	0	0	10	10	—	—
KBPPK	0010.20	5	4-1	56	46	0	0	16	16	—	—
KNNNK	0009.00/30	5	4-1	44	180	0	0	21	21	—	—
KNNPK	0002.10	5	4-1	194	296	0	0	15	15	—	—
KNPPK	0001.20	5	4-1	2	5	0	0	12	12	—	—
KPPPK	0000.30	5	4-1	11	35	0	0	11	11	—	—
KQBBK	1020.00	5	4-1	182	673	0	0	6	6	—	—
KQBNK	1011.00	5	4-1	54,680	236,453	0	0	4	4	—	—
KQBPk	1010.10	5	4-1	68	255	0	0	6	6	—	—
KQNNK	1002.00	5	4-1	182	673	0	0	7	7	—	—
KQNPk	1001.10	5	4-1	11,789	56,328	0	0	5	5	—	—
KQPPK	1000.20	5	4-1	1,264	4,476	0	0	6	6	—	—
KQQBK	2010.00	5	4-1	96,576	412,131	0	0	3	3	—	—
KQQNK	2001.00	5	4-1	13	58	0	0	4	4	—	—
KQQPK	2000.10	5	4-1	138	732	0	0	4	4	—	—
KQQQK	9000.00/30	5	4-1	1,513	6,553	0	0	3	3	—	—
KQQRK	2100.00	5	4-1	56,174	218,959	0	0	3	3	—	—
KQRBK	1110.00	5	4-1	1,198	5,865	0	0	4	4	—	—
KQRNK	1101.00	5	4-1	7,474	31,526	0	0	4	4	—	—
KQRPk	1100.10	5	4-1	3	15	0	0	5	5	—	—
KQRRK	1200.00	5	4-1	18	87	0	0	4	4	—	—
KRBBK	0120.00	5	4-1	24	126	0	0	10	10	—	—
KRBNK	0111.00	5	4-1	8,391	26,677	0	0	7	7	—	—
KRBPk	0110.10	5	4-1	1	5	0	0	8	8	—	—
KRNNK	0102.00	5	4-1	602	2,052	0	0	10	10	—	—
KRNPk	0101.10	5	4-1	579	1,436	0	0	8	8	—	—
KRPPK	0100.20	5	4-1	4	24	0	0	8	8	—	—
KRRBK	0210.00	5	4-1	4,761	17,210	0	0	5	5	—	—
KRRNK	0201.00	5	4-1	8,533	29,009	0	0	5	5	—	—
KRRPK	0200.10	5	4-1	16	56	0	0	6	6	—	—
KRRRK	0900.00/30	5	4-1	3,566	13,290	0	0	4	4	—	—

Table 1c. Chess Endgames: 3-to-5-man DTC data.

Endgame		DTZ Metric									
		# of maximal positions						max depth, moves			
		1-0		0-1		1-0		0-1			
GBR	# w-b	wtm	btm	wtm	btm	wtm	btm	wtm	btm	wtm	btm
KPK	0000.10	3	2-1	8	4	0	0	10	10	—	—
KBKP	0010.01	4	2-2	104	28	779	585	1	0	3	4
KNKP	0001.01	4	2-2	23	6	6	2	6	5	8	8
KPKP	0000.11	4	2-2	1	1	1	1	11	10	10	11
KQKP	1000.01	4	2-2	1	1	20	385,976	26	26	1	1
KRRP	0100.01	4	2-2	2	38	3	3	13	12	10	10
KBPk	0010.10	4	3-1	38	42	0	0	13	13	—	—
KNPK	0001.10	4	3-1	108	8	0	0	13	13	—	—
KPPK	0000.20	4	3-1	125	152	0	0	7	7	—	—
KQPK	1000.10	4	3-1	25	107	0	0	3	3	—	—
KRPK	0100.10	4	3-1	1,643	6,556	0	0	3	3	—	—
KBBKP	0020.01	5	3-2	16	16	5	47	21	21	8	8
KBNKP	0011.01	5	3-2	202	39	494	157	20	20	8	8
KBPk	0040.10	5	3-2	13	22	508	1,524	25	25	2	3
KBPKN	0013.10	5	3-2	20	5	23	86	30	30	3	4
KBPkP	0010.11	5	3-2	9	4	24	30	37	37	5	6
KBPkQ	3010.10	5	3-2	1,438	30	1	2	3	3	42	42
KBPkR	0310.10	5	3-2	5	39	5	6	13	12	18	19
KNNKP	0002.01	5	3-2	18	13	1	1	82	81	11	11
KNPKB	0031.10	5	3-2	39	33	5	18	24	24	8	9
KNPKN	0004.10	5	3-2	2	25	27	132	30	29	3	4
KNPKP	0001.11	5	3-2	1	1	12	4	23	23	7	7
KNPKQ	3001.10	5	3-2	2,459	4	1	1	3	3	43	43
KNPKR	0301.10	5	3-2	8	36	3	9	18	18	39	40
KPPKB	0030.20	5	3-2	2	5	1	13	12	12	1	2
KPPKN	0003.20	5	3-2	3	8	45	100	14	13	6	7
KPPkP	0000.21	5	3-2	1	3	1	4	21	21	7	7
KPPkQ	3000.20	5	3-2	8	15	19	16	6	5	29	29
KPPkR	0300.20	5	3-2	67	83	13	14	14	14	15	15
KQBKP	1010.01	5	3-2	5	14	791	2,934,215	11	11	1	1
KQNKP	1001.01	5	3-2	7	1	928	5,722,853	17	17	1	1
KQPKB	1030.10	5	3-2	13,462	65,629	374	1,290	5	5	1	2
KQPKN	1003.10	5	3-2	26	105	3	9	6	6	1	2
KQPKP	1000.11	5	3-2	69	2	1,024	7,412,631	5	5	1	1
KQPKQ	4000.10	5	3-2	1	3	2	4	71	70	15	16
KQPKR	1300.10	5	3-2	3	19	5,177	26,128	17	17	2	3
KQQKP	2000.01	5	3-2	13,425	1,987	140	16,368	3	3	1	1
KQRKP	1100.01	5	3-2	76,181	2,592	683	892,287	3	3	1	1
KRBKP	0110.01	5	3-2	2	10	4	8	11	11	4	5
KRNKP	0101.01	5	3-2	19	26	2	2	15	14	10	10
KRPk	0130.10	5	3-2	5	7	502	1,672	53	53	1	2
KRPkN	0103.10	5	3-2	8	15	4	12	31	31	1	2
KRPkP	0100.11	5	3-2	20	22	17	18	9	9	10	10
KRPkQ	3100.10	5	3-2	2	5	3	1	9	8	75	76
KRPkR	0400.10	5	3-2	3	4	14	43	35	35	6	7
KRRKP	0200.01	5	3-2	16	32	988	1,506,491	9	9	1	1
KBBPK	0020.10	5	4-1	5	1	0	0	12	12	—	—
KBNPK	0011.10	5	4-1	74	199	0	0	5	5	—	—
KBPkP	0010.20	5	4-1	16	32	0	0	9	9	—	—
KNNPK	0002.10	5	4-1	6,992	7,623	0	0	8	8	—	—

Table 2a. Chess Endgames: 3-to-5-man DTZ data.

Endgame				DTZ Metric							
				# of maximal positions				max depth, moves			
								1-0		0-1	
				GBR	#	w-b	wtm	btm	wtm	btm	wtm
KNPPK	0001.20	5	4-1	1	7	0	0	6	6	—	—
KPPPK	0000.30	5	4-1	16	64	0	0	7	7	—	—
KQBPk	1010.10	5	4-1	2,085	6,415	0	0	3	3	—	—
KQNPk	1001.10	5	4-1	958	4,181	0	0	3	3	—	—
KQPPK	1000.20	5	4-1	20	88	0	0	3	3	—	—
KQQPK	2000.10	5	4-1	29	81	0	0	3	3	—	—
KQRPk	1100.10	5	4-1	2,330	6,022	0	0	3	3	—	—
KRBPk	0110.10	5	4-1	67	114	0	0	4	4	—	—
KRNPK	0101.10	5	4-1	36	152	0	0	4	4	—	—
KRPPK	0100.20	5	4-1	270	651	0	0	3	3	—	—
KRRPK	0200.10	5	4-1	6,122	11,124	0	0	3	3	—	—

Table 2b. Chess Endgames: 3-to-5-man DTZ data.

Endgame				DTZ ₅₀ Metric							
				# of maximal positions				max depth, moves			
				1-0		0-1		1-0		0-1	
GBR	#	w-b	wtm	btm	wtm	btm	wtm	btm	wtm	btm	
KBBKN	0023.00	5	3-2	347,796	485,538	44	222	50	50	0	1
KBBKP	0020.01	5	3-2	16	16	3	4	21	21	9	10
KBBKQ	3020.00	5	3-2	248	58	86,896	24,793	4	3	50	50
KBNKN	0014.00	5	3-2	12,123	5,857	104	533	50	50	0	1
KBNKP	0011,01	5	3-2	202	39	494	157	20	20	8	8
KBPKN	0013.10	5	3-2	20	5	23	86	30	30	3	4
KNNKP	0002.01	5	3-2	60,080	12,023	1	1	50	50	11	11
KNNKQ	3002.00	5	3-2	2,387	465	6,352	2,010	1	0	50	50
KNPKN	0004.10	5	3-2	2	25	27	132	30	29	3	4
KNPKQ	3001.10	5	3-2	2,459	4	1	1	3	3	43	43
KPPKP	0000.21	5	3-2	1	3	1	4	21	21	7	7
KPPKQ	3000.20	5	3-2	8	15	19	16	6	5	29	29
KQPKP	1000.11	5	3-2	69	2	1,024	7,412,631	5	5	1	1
KQPKQ	4000.10	5	3-2	1,595	2,415	2	4	50	50	15	16
KQRPK	1100.01	5	3-2	76,181	2,592	683	892,287	3	3	1	1
KQRKQ	4100.00	5	3-2	23	156	1	2	50	50	8	9
KRBKR	0410.00	5	3-2	1,041	175	3	14	50	50	3	4
KRPKB	0130.10	5	3-2	130	254	502	1,672	50	50	1	2
KRPKP	0100.11	5	3-2	20	22	17	18	9	9	10	10
KRPKQ	3100.10	5	3-2	2	5	9,275	4,898	9	8	50	50

Table 3. Chess Endgames: 3-to-5-man data where $EZ_{50} \neq EZ$.

Endgame				DTZ Metric							
				# of maximal positions				max depth, moves			
				1-0		0-1		1-0		0-1	
				wtm	btm	wtm	btm	wtm	btm	wtm	btm
GBR	#	w-b									
KBBKNN	0026.00	6	3-3	11	1	488	1,518	38	38	3	4
KQKQBB	2060.00	6	3-3	984	5,128	137	714	6	6	3	4
KQKQNN	2006.00	6	3-3	2	8	1	36,110	7	7	1	1
KQKQQR	5300.00	6	4-2	4	2	1	12	48	47	56	56
KRRKRB	0530.00	6	3-3	22	13	1	455	54	54	6	6
KBBBKN	0093.00/30	6	4-2	6	6	951	4,838	12	12	0	1
KBBBKQ	3090.00/30	6	4-2	1	9	1	3	10	9	51	51
KBBNKN	0024.00	6	4-2	9	54	3,663	18,984	31	31	0	1
KBNNKN	0015.00	6	4-2	17	56	4,335	22,890	28	28	0	1
KBNNKQ	3012.00	6	4-2	5	1	1	4	12	11	49	49
KNNNKQ	3009.00/30	6	4-2	1	1	6	11	9	8	35	35
KQNNKQ	4002.00	6	4-2	2	2	5	20	71	71	13	14
KRNKNQ	3102.00	6	4-2	2	1	2	3	28	27	41	41

Table 4. Chess Endgames: some 6-man DTZ data.

Endgame				DTZ ₅₀ Metric							
				# of maximal positions				max depth, moves			
				1-0		0-1		1-0		0-1	
				wtm	btm	wtm	btm	wtm	btm	wtm	btm
GBR	#	w-b									
KBBKNN	0026.00	6	3-3	46	17	488	1,518	29	28	3	4
KQKQBB	2060.00	6	3-3	1	5	137	714	8	8	3	4
KQKQNN	2006.00	6	3-3	2	8	1	36,110	7	7	1	1
KQKQQR	5300.00	6	4-2	4	2	6	26	48	47	50	50
KRRKRB	0530.00	6	3-3	372	107	1	455	50	50	6	6
KBBBKN	0093.00/30	6	4-2	3	6	951	4,838	14	14	0	1
KBBBKQ	3090.00/30	6	4-2	1	9	11	15	10	9	50	50
KBBNKN	0024.00	6	4-2	9	54	3,663	18,984	31	31	0	1
KBNNKN	0015.00	6	4-2	3	3	4,335	22,890	29	29	0	1
KBNNKQ	3012.00	6	4-2	5	1	1	4	12	11	49	49
KNNNKQ	3009.00/30	6	4-2	1	1	6	11	9	8	35	35
KQNNKQ	4002.00	6	4-2	10,534	9,796	5	20	50	50	13	14
KRNKNQ	3102.00	6	4-2	2	1	2	3	28	27	41	41

Table 5. Chess Endgames: some 6-man DTZ₅₀ data.

The following lines, starting from some positions listed in Table 7 below, show strategies variously retaining the win, failing to retain the win, repeating positions to draw or being suboptimal. They include an established notation showing the criticality of the moves:

" \equiv unique value-preserving move; ' \equiv only optimal move; ° \equiv only legal move.

KBBKP position BB-P1 – $dtz = 1m$; $dtz_{50} = 7m$:

$S\phi^- S\sigma, \sigma = C^-, M^-$ or Z^- : 1. ... a1Q+?? { $dtz = 51m$; White can force a 50m draw} $\frac{1}{2}$ - $\frac{1}{2}$.

$SZ_{50}^+ - SZ_{50}^-$: 1. ... Kc4" 2. Bf3+ Kc3 3. Be1+' Kd4" 4. Bf2+' Ke5' 5. Bg3+' Kf6' 6. Bh4+' Kg7' { $dtm = 17m$ } 0-1.

KNNKP position NN-P1 – $dtz = 20m$, $dtc = 63m$, $dtm = 64m$, $dtz_{50} = 44m$:

$S(C, M, Z)^- S^- SZ_{50}^+$: 1. Ng1?? h3" { $dtz = 61m$; Black can force a 50m draw} $\frac{1}{2}$ - $\frac{1}{2}$.

$SZ_{50}^- - SZ_{50}^+$: 1. Ngf2' Ke3' 2. Kc3' Ke2' 3. Kd4' Kd2' 4. Ne4+' Ke2' 5. Neg5' Kd2' 6. Nf3+' Ke2' 7. Ke4' Kf1' 8. Kd3 Kg2° 9. Nfg5' Kg3' 10. Ke3 Kg4' 11. Ke4' Kg3' ... 1-0.

KNNKP position NN-P2 – $dtz = 1m$, $dtz_{50} = 43m$:

SZ – **Sr**: 1. **Nbc4**?? { $dtz = 58m$; Black can force a 50m draw} $\frac{1}{2}$ - $\frac{1}{2}$.

S(C/M) – **SZ**₅₀⁺: 1. Na4" { $dtz_{50} = 42m$, $dtm = 88m$ } Kd2° 1-0.

SZ₅₀⁻ – **SZ**₅₀⁺: 1. Na4" Kd2° 2. Nc4+ Kd3' 3. Ncb2+ Kd2 4. Kb1 Ke3' 5. Kc1 Ke2 6. Kc2' Ke3' 7. Kc3' Ke4' 8. Nd3 Ke3 9. Ndc5' Kf4' 10. Kd4' Kf5' 11. Nd3 Ke6 12. Ke4' Kd6 13. Nf4 Kc6 14. Ne2' Kd6' 15. Nd4' Ke7' 16. Ke5' Kf7' 17. Kf5' Ke7 18. Nb5' Kd7 19. Ke5' Kc6' 20. Na3' Kd7' 21. Nc5+ Kc7' 22. Ne4' Kf7' 23. Kd6' Kg7 24. Ke6' Kg6' 25. Nc4' Kg7' 26. Ned2 Kg6 27. Nf3' Kh6 28. Kf5' Kg7 29. Ng5' Kf8 30. Kf6' Ke8' 31. Ke6' Kf8' 32. Nh3' Kg8' 33. Nf4 Kg7' 34. Ke7' Kh6 35. Kf6' Kh7' 36. Ne2 Kh6 37. Ng3' Kh7' 38. Nf5' Kg8 39. Ke7' Kh8' 40. Ne5 Kh7 41. Ke8 Kg8 42. Ng6' Kh7' 43. Kf7' a4° { $dtm = 3m$ } 1-0.

KNNKP position NN-P3 – $dtz = 1m$, dtz_{50} indicates 'draw', $dtr = 51m$, $dtz_{51} = 31m$:

SZ^o – **Sφ**, **σ** = **C**, **M**, **Z** or **Z**₅₀⁺: 1. **Kc2**? { $dtr > 51m$ }.

SZ^o**H**₅₀⁻ – **SH**₅₀⁺: 1. Nb1+ { $dtr = 51m$, controlling DTR} Ka4'

KNNKP position NN-P4 – $dtz = 16m$, dtz_{50} indicates 'draw', $dtz_{51} = 25m$, $mleft = 25m$:

SZ^o**H**₅₀⁻ – **SH**₅₀⁺: 1. Nd5+? { $dtz_{51} = 26m$ } Kc4' 2. Ndc3 Kb4' {NN-P4 repeated} $\frac{1}{2}$ - $\frac{1}{2}$.

KQPKQ position QP-Q1 - $dtr = 52m$, $dtz = 1m$, $dtz_{50} = 50m$:

S – **Sr**, **σ** = **C**, **M** or **Z**: 1. **b7**?? { $dtz = 51m$; Black can force a 50m draw} $\frac{1}{2}$ - $\frac{1}{2}$.

SZ₅₀⁻ – **SZ**₅₀⁺: 1. Qg5" Qe4' 2. Kc5" Qc2+ 3. Kd5 Qb3+ 4. Kc6' Qe6+ 5. Kc5' Qc8+ 6. Kd4' Qh8+ 7. Kc4' Qh7 8. Qd5' Qc2+ 9. Kb4 Qb2+ 10. Kc5' Qa3+ 11. Kc6' Qa4+ 12. Kd6 Qf4+ 13. Kd7' Qg4+ 14. Qe6' Qg7+ 15. Kd6' Qg3+ 16. Kc5' Qg5+ 17. Kc4 Qc1+ 18. Kd5' Qb2' 19. Qg6' Qb5+ 20. Kd4" Qb4+ 21. Ke5' Qc5+ 22. Kf4" Qd4+ 23. Kf5 Qc5+ 24. Kg4' Qd4+ 25. Kh5' Qd5+ 26. Kh6' Ke1' 27. Qg1+ Ke2' 28. Qg4+ Kf1' 29. Qg5' Qc6+ 30. Qg6" Qb7' 31. Qf6+ Ke2' 32. Kg5' Ke3' 33. Qe5+ Kf2' 34. Qc5+ Ke2' 35. Kf4' Qf3+ 36. Ke5' Qg3+ 37. Ke6' Qh3+ 38. Kd6' Qh6+ 39. Kc7' Qg7+ 40. Kc6' Qf6+ 41. Qd6' Qc3+ 42. Kd7' Qf3' 43. Kc8 Qc3+ 44. Kd8' Qa5' 45. Ke7' Qb5 46. Qf6' Qb1 47. Kf7 Qh7+ 48. Kf8' Qb1 49. Qe7+ Kd1 50. b7' { $dtm = 21m$ } 1-0.

KRPKP position RP-P2 – $dtz = 1m$, $dtz_{50} = 6m$:

Sφ – **Sr**, **σ** = **C**, **M** or **Z**: 1. ... **g1Q**?? { $dtr > 50m$; White can force a 50m draw} $\frac{1}{2}$ - $\frac{1}{2}$.

SZ₅₀⁺ – **SZ**₅₀⁺: 1. ... Kb2" 2. Rb4+ Kc2" 3. Rc4+ Kd2' 4. Rd4+ Ke2' 5. Re4+ Kf2" 6. Re7 g1Q" { $dtm = 49m$ } 0-1.

KRPKP position RP-Q1 – $dtz = 2m$, $dtz_{50} = 21m$:

Sφ – **Sr**, **σ** = **C** or **M**: 1. ... **Qd6**?? { $dtr > 50m$; White can force a 50m draw} $\frac{1}{2}$ - $\frac{1}{2}$.

Sφ – **SZ**: 1. ... **Qe4**?? { $dtr > 50m$; White can force a 50m draw} $\frac{1}{2}$ - $\frac{1}{2}$.

SZ₅₀⁺ – **SZ**₅₀⁺: 1. ... Qe6+ 2. Kg5' Qg8+ 3. Kh6' Qd5' 4. Rg7' Qh1+ 5. Kg6' Qg1+ 6. Kf7' Qf1 7. Rg6+ Kb7' 8. Rf6' Qg2 9. Ke6 Qe4+ 10. Kd6 Kb6 11. Rf7' Kb5' 12. Rf6 Kc4' 13. Rf7' Kd4' 14. Rf8' Qd5+ 15. Ke7' Qc5+ 16. Kf7 Kd5" 17. Kg7 Qg1+ 18. Kf6 Qg4' 19. Ke7' Qe6+ 20. Kd8° Kc6 21. Rf6 Qxf6+ { $dtm = 2m$ } 0-1.

KBBKNN position BB-NN1 - $dtz = 1m$, $dtz_{50} = 28m$:

Sr – **Sφ**, **σ** = **C**, **M** or **Z**: 1. **Bxg6**?? { $dtz = 54m$; Black can force a 50m draw} $\frac{1}{2}$ - $\frac{1}{2}$.

SZ₅₀⁻ – **SZ**₅₀⁺: 1. Bd6" Nh8' 2. Bc6+ Ka5° 3. Kb3" Nc1+ 4. Kc4" Nf7' 5. Bc7+ Ka6° 6. Bd5" Nh8' 7. Bf3' Ng6' 8. Bd6" Nh4' 9. Be4" Ne2' 10. Bh2" Ka5' 11. Bc7+ Ka6' 12. Kc5' Ka7' 13. Bd3' Ng1' 14. Bg3 Ng2' 15. Kc6' Nh3' 16. Bf1' Nh4' 17. Bf2+ Kb8' 18. Bb6' Ka8' 19. Ba6' Kb8' 20. Bc4' Nh5' 21. Bc7+ Ka7 22. Be5' Nh4' 23. Bd6' Nh5 24. Kc7' Nf6' 25. Bc5+ Ka8° 26. Bb5 Nd5+ 27. Kc8" Ne1 28. Bc6# 1-0.

KQQKBB position QQ-BB1 - $dtz = 2m$, $dtz_{50} = 7m$:

SZ – **Sφ**: 1. **Qxd4**?? **Bxd4** { $dtz = 67m$; Black can force a 50m draw} $\frac{1}{2}$ - $\frac{1}{2}$.

SZ₅₀⁻ – **SZ**₅₀⁺: 1. Kb1" Be4+ 2. Ka2" Bd5+ 3. Ka3' Bd6+ 4. Ka4' Bc6+ 5. Ka5 Kc3 6. Qc1+ Kb3 7. Qxc6' { $dtm = 2m$ } 1-0.

KBNNKQ position BNN-Q1 - $dtz = 1m$, $dtz_{50} = 36m$:

$S\phi - S\sigma$, $\sigma = C$, M or Z : 1. ... Qxa1??? { $dtz = 52m$; Black can force a 50m draw} $\frac{1}{2}$ - $\frac{1}{2}$.

$SZ_{50}^- - SZ_{50}^+$: 1. ... Qh7+" 2. Kd2' Qd7+" 3. Kc3' Ke2' 4. Bb2' Qg4" 5. Kb3' Qe6" 6. Kc3' Qe4" 7. Kb3' Qg4' 8. Kc3' Qf4' 9. Kb3' Qb8+" 10. Kc2' Qb4' 11. Na3' Qe4+" 12. Kb3' Qd5+" 13. Kc3' Qf3+" 14. Kc4' Kd1 15. Kb4' Qb7+" 16. Nb5' Kc2' 17. Bd4' Qe7+" 18. Kc4' Qe6+" 19. Kc5' Qf5+" 20. Kc4' Qc8+" 21. Kb4' Qf8+" 22. Ka4 Qg8' 23. Kb4 Kd3' 24. Bc3' Qd5' 25. Bd4 Qc4+' 26. Ka5' Qg8' 27. Ka4' Qa8+' 28. Kb4' Qf8+' 29. Kb3' Qe7' 30. Bb2' Qe6+' 31. Ka4 Qa2+ 32. Ba3' Qc4+' 33. Ka5 Qd5' 34. Kb4' Qe4+ 35. Ka5 Qa8+' 36. Kb6 Qxh8 { $dtm = 22m$ } 0-1.

KQNNKQ position QNN-Q1 - $dtz = 3m$, $dtz_{50} = 4m$, $dtm = 5m$:

$SZ^- - SZ^+$: 1. Qa3+'?? Kd1' 2. Qa1+" Ke2° 3. Qxh1" { $dtz = 52m$ } $\frac{1}{2}$ - $\frac{1}{2}$.

$SZ_{50}^- - SZ_{50}^+$: 1. Qe3+" Kbl' 2. Qb6+" Kcl' 3. Qb2+' Kd1° 4. Qd2# 1-0.

Endgame	res.	# extra draws		# delayed		% of nominal wins			
		wtm	btm	wtm	btm	wtm	btm	wtm	btm
KBBKN	1-0	3,993,656	7,852,543	0	0	21.05	48.20	0	0
KBBKP	1-0	171	687	3,889	1,800	ϵ	ϵ	0.01	ϵ
	0-1	119,226	1,444,441	1,524	3,741	5.85	8.47	0.07	0.02
KBBKQ	0-1	2,154,114	490,797	0	0	8.49	1.46	0	0
KBNKN	1-0	139,893	72,483	0	0	0.52	1.93	0	0
KBNKP	1-0	185	275	1,641	1,685	ϵ	ϵ	ϵ	ϵ
KBPKN	1-0	257	264	602	1,530	ϵ	ϵ	ϵ	ϵ
KNNKP	1-0	10,684,968	9,495,721	17,093,973	6,239,778	26.35	46.87	42.16	30.80
	0-1	4,255	10,877	301	357	0.14	0.06	0.01	ϵ
KNNKQ	0-1	11,990	3,667	0	0	0.05	0.01	0	0
KNPKN	1-0	61	86	48	39	ϵ	ϵ	ϵ	ϵ
KNPKQ	0-1	1	0	0	0	ϵ	0	0	0
KPPKP	1-0	1,834	2,062	149	55	ϵ	ϵ	ϵ	ϵ
KPPKQ	1-0	1,641	3	0	0	0.01	0.01	0	0
KQPKP	1-0	19	3,266	2,664	2,207	ϵ	ϵ	ϵ	ϵ
KQPKQ	1-0	28,468	22,411	42,756	28,526	0.02	0.08	0.03	0.10
KQRKP	1-0	0	79	0	0	0	ϵ	0	0
KQRKQ	1-0	230	1,106	0	0	ϵ	ϵ	0	0
KRBKR	1-0	2,263	725	0	0	0.01	0.02	0	0
KRPKB	1-0	35	83	53	74	ϵ	ϵ	ϵ	ϵ
KRPKP	1-0	0	240	124	33	0	ϵ	ϵ	ϵ
	0-1	679	12,137	26	30	0.14	0.05	0.01	ϵ
KRPKQ	1-0	1,592	1	116	0	ϵ	ϵ	ϵ	0
	0-1	72,802	29,723	26,336	9,097	0.06	0.02	0.02	ϵ
KBBKNN	1-0	141,874,223	38,562,549	4,961,624	1,402,773	50.15	70.98	1.75	2.58
KQQKBB	1-0	23,343	6,776,509	1,244,572	5,432,160	ϵ	0.58	0.18	0.47
KQQKNN	1-0	130	44,687	4,704	22,000	ϵ	ϵ	ϵ	ϵ
KQQKQR	0-1	17,313	41,775	42,552	66,504	0.02	0.01	0.04	0.01
KRRKRB	1-0	380	145	0	0	ϵ	ϵ	0	0
	0-1	396	11,281	30	799	0.02	0.03	ϵ	ϵ
KBBBKN	1-0	743,762	37,035,833	55,589,963	161,070,140	0.15	6.16	11.28	26.80
KBBBKQ	0-1	21,650,797	31,223,711	6,004,068	11,096,464	15.04	6.15	4.17	2.19
KBBNKN	1-0	640,358	36,582,112	136,891,517	318,970,567	0.03	1.74	6.44	15.17
KBNKNK	1-0	96,123	1,016,653	10,322,215	13,062,956	ϵ	0.05	0.46	0.70
KBNNKQ	0-1	178,774	178,631	179,015	143,015	0.03	0.01	0.03	0.01
KNNNKQ	0-1	125,488	181,848	91,063	99,907	0.09	0.04	0.07	0.02
KQNNKQ	1-0	49,329	38,050	0	0	ϵ	0.01	0	0
	0-1	1,538	206,733	0	2	0.04	0.05	0	ϵ
KRNNKQ	0-1	33,448	252,183	10,270	30,764	0.04	0.03	0.01	ϵ

Table 6. The impact of the 50-move drawing rule.

Key	Position	stm	depth in plies					Notes
			dte	dtm	dtr	dtz	dtz ₅₀	
EZ ₅₀ ≠ EZ								
BB-N	1-0 8/8/7B/4k3/4B3/3K4/1n6	w	119	143	119	119	—	q.v. BB-P
BB-P	1-0 8/8/7B/4k3/4B3/1p1K4/8	b	6	144	119	6	—	1. ... b1=N+ {BB-N}
	0-1 8/8/6B1/3K4/5B2/8p7/3k4	b	1	157	136	1	—	1. ... a1=Q" {BB-Q}
BB-Q	0-1 8/8/6B1/3K4/5B2/8/8/q2k4	w	136	156	136	136	—	q.v. BB-P
BN-N	1-0 8/8/3K4/8/8/3B4/k7/1n1N4	w	139	199	139	139	—	q.v. BN-P
BN-P	1-0 8/8/3K4/8/8/3B4/kp6/3N4	b	9	200	139	9	—	1. ... b1=N {BN-N}
BP-N	1-0 1n6/3P4/8/8/1K6/7B/8/k7	w	1	199	138	1	—	1. d8=N" {dtz=138p}
NN-P	1-0 K1k5/3N1N2/8/8/4p3/8/8/8	w	169	169	164	164	—	maxDTZ pos.
	0-1 3k3N/3N4/3K4/8/8/8/7p/8	b	1	145	126	1	—	1. ... h1=Q" {NN-Q}
NN-Q	0-1 3k3N/3N4/3K4/8/8/8/8/7q	w	126	144	126	126	—	q.v. NN-P
NP-N	1-0 kn6/3P4/1K6/8/8/8/3N4/8	w	1	191	130	1	—	1. d8=B" {dtz=130p}
NP-Q	0-1 1k1K4/4P1N1/8/8/8/6q1/8/8	w	6	124	103	6	—	1. e8=N {dtz=103p}
PP-P	1-0 8/4P3/8/8/8/4P3/kp1K4/8	b	2	244	102	2	—	1. ... b1=Q {PP-Q}
PP-Q	1-0 8/4P3/8/8/8/4P3/k2K4/1q6	w	1	243	102	1	—	1. e8=Q" {QP-Q}
QP-P	1-0 8/4Q3/8/8/8/K7/6Pp/5k2	w	5	191	?	1	—	
QP-Q	1-0 4Q3/8/8/8/8/4P3/k2K4/1q6	b	222	242	102	102	—	q.v. PP-Q
QR-P	1-0 Q7/2k5/8/8/8/8/R2p4/K7	b	2	134	119	2	—	1. ... d1=Q {QR-Q}
QR-Q	1-0 Q7/2k5/8/8/8/8/R7/K2q4	w	119	133	119	119	—	q.v. QR-P
RB-R	1-0 8/3B4/8/1R6/5r2/8/3K4/5k2	w	117	129	117	117	—	maxDTZ pos.
RP-B	1-0 K1R5/8/3k4/3P4/8/8/1b6/8	w	113	131	105	105	—	maxDTZ pos.
RP-P	1-0 6R1/P6k1/k6/8/8/8/3p4/8	b	1	136	120	1	—	1. ... d1=Q" {dtz=120p}
	0-1 8/8/8/5PR1/8/2K5/5p2/k7	w	2	188	130	2	—	1. Kd4" f1Q" {dtz=130p}
RP-Q	1-0 6R1/P7/2q5/2k5/8/8/8/6K1	b	2	118	102	2	—	only frustrated btm 1-0 pos.
	0-1 8/7R/6K1/8/5P2/8/8/k6q	b	116	165	107	3	—	
BB-NN	1-0 8/6B1/8/8/2B1n3/6K1/3k3n/8	w	1	147	122	1	—	1. Kxh2" {dtz=122p}
QQ-BB	1-0 8/8/8/4b3/8/Q7/2k1b3/K5Q1	w	2	143	121	2	—	1. Qc3" Bxc3+ {dtz=121p}
QQ-NN	1-0 8/8/8/1Q6/3n4/2n3k1/K3Q3	w	3	135	113	3	—	1. Kb1" Ncxe1 {dtz=113p}
QQ-QR	0-1 8/Q7/1Q6/8/r7/8/8/qK5k	w	2	132	116	2	—	1. Kc2° Rxa7" {dtz=116p}
RR-RB	1-0 3R4/8/R7/8/8/8/6r1/k3K2b	b	102	122	102	102	—	
	0-1 8/R7/8/4b3/8/1r6/R7/K3k3	w	2	116	102	2	—	1. Rb2° Rxb2" {dtz=102p}
BBB-N	1-0 8/8/8/8/8/2B1n3/K1k3BB	w	2	145	119	2	—	1. Bb6 Kxc2 {dtz=119p}
BBB-Q	0-1 8/8/8/8/q7/3BB3/8/K2kb3	w	2	142	120	2	—	1. Kb2 Kxe1 {dtz=120p}
BBN-N	1-0 8/8/8/8/8/2N2B2/K1kn3B	w	2	141	115	2	—	1. Bb6 Kxc2 {dtz=115p}
BNN-N	1-0 n7/8/8/8/8/6B1/6N1/K4kn1	w	2	181	119	2	—	1. Ne3+" Kxg1° {dtz=119p}
BNN-Q	0-1 q7/8/8/8/8/N7/3N4/K1kb4	w	2	126	104	2	—	1. Ndc4 Kxd1" {dtz=104p}
NNN-Q	0-1 8/8/2q5/8/8/N7/3N4/K1kn4	w	2	126	104	2	—	1. Ndc4 Kxd1" {dtz=104p}
QNN-Q	1-0 7q/1Q6/8/5N2/8/8/8/K1k4N	w	101	107	101	101	—	1. Ng7" ...
	0-1 8/8/8/1N6/8/8/N7/kqK2Q2	w	2	124	104	2	—	1. Kd2° Qxf1" {dtz=104p}
RNN-Q	0-1 8/8/1R6/q7/3N4/8/4N3/K2k4	w	2	122	102	2	—	1. Kb2 Qxb6+" {dtz=102p}
Strategy Failure Positions								
BB-P1	1-0 8/8/8/1k6/8/8/p4BB1/3K4	b	1	123	58?	1	13	S(Cσ/M/Z)σ ×
NN-P1	1-0 8/8/8/2K3Np/7N/3k4/8	w	126	127	40	88	88	S(C/M/Z)σ ×
NN-P2	1-0 8/8/1N6/p7/8/4N3/8/K1k5	w	176	177	100?	2	86	SZσ ×; S(C/M) ok
NN-P3	1-0 8/8/8/2pN4/8/k1N5/8/2K5	w	115	115	102	2	—	SZ°H ₅₀ : 1. Nb1+
NN-P4	1-0 8/8/8/2p5/1k6/2N5/2K5/1N6	w	113	113	102	32	—	SZ°H ₅₀ repeats positions
QP-Q1	1-0 8/8/1P5Q/1K6/3q4/8/5k2/8	w	103	125	99	1	99	S(C/M/Z)σ ×
RP-P1	1-0 6R1/8/Pk6/8/8/8/p2K4/8	w	3	31	26?	1	5	S(C/Z)σ ×; SM ok
RP-P2	1-0 8/8/5K2/8/2R2P2/8/6p1/k7	b	1	159	?	1	11	S(C/M/Z)σ ×
RP-Q1	1-0 8/4q2R/k5K1/8/5P2/8/8/8	b	113	163	?	3	41	S(C/M/Z)σ ×
BB-NN1	1-0 8/8/6n1/8/k3BB2/8/n1K5/8	w	1	133	55	1	55	S(C/M/Z)σ ×
QQ-BB1	1-0 8/Q7/8/3bb3/8/8/3k4/K4Q2	w	3	17	13	3	13	SZ° ×
BNN-Q1	0-1 7N/6q1/8/8/2N5/3K1k2/8/B7	b	1	125	?	1	71	S(C/M/Z)σ ×
QNN-Q1	0-1 8/2N5/8/2q5/5N2/2k5/8/2K4Q	b	5	9	7	5	7	S(C/Z)σ ×; SM ok

Table 7. Example Positions.⁷⁷ Without a DTR EGT, it is not always possible to determine *dtr* precisely.